EXERCISE- 9.1

Question 1:

Write the first five terms of the sequences whose n^{th} term is $a_n = n(n+2)$

$a_n = n(n+2)$

Substituting n = 1, 2, 3, 4, and 5, we obtain

$$a_1 = 1(1+2) = 3$$
$$a_2 = 2(2+2) = 8$$
$$a_3 = 3(3+2) = 15$$
$$a_4 = 4(4+2) = 24$$
$$a_5 = 5(5+2) = 35$$

Therefore, the required terms are 3, 8, 15, 24, and 35.

Question 2:

Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{n}{n+1}$

$a_n = \frac{n}{n+1}$

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = \frac{1}{1+1} = \frac{1}{2},\ a_2 = \frac{2}{2+1} = \frac{2}{3},\ a_3 = \frac{3}{3+1} = \frac{3}{4},\ a_4 = \frac{4}{4+1} = \frac{4}{5},\ a_5 = \frac{5}{5+1} = \frac{5}{6}$$

Therefore, the required terms are $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$, and $\frac{5}{6}$.

Question 3:

Write the first five terms of the sequences whose n^{th} term is $a_n = 2^n$

$a_n = 2^n$

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = 2^1 = 2$$
$$a_2 = 2^2 = 4$$
$$a_3 = 2^3 = 8$$
$$a_4 = 2^4 = 16$$
$$a_5 = 2^5 = 32$$

Therefore, the required terms are 2, 4, 8, 16, and 32.

Question 4:

Write the first five terms of the sequences whose n^{th} term is $a_n = \frac{2n-3}{6}$

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = \frac{2\times1-3}{6} = \frac{-1}{6}$$
$$a_2 = \frac{2\times2-3}{6} = \frac{1}{6}$$
$$a_3 = \frac{2\times3-3}{6} = \frac{3}{6} = \frac{1}{2}$$
$$a_4 = \frac{2\times4-3}{6} = \frac{5}{6}$$
$$a_5 = \frac{2\times5-3}{6} = \frac{7}{6}$$

Therefore, the required terms are $\frac{-1}{6}, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}$, and $\frac{7}{6}$.

Question 5:

Write the first five terms of the sequences whose n^{th} term is $a_n = (-1)^{n-1} 5^{n+1}$

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Therefore, the required terms are 25, –125, 625, –3125, and 15625.

Question 6:

Write the first five terms of the sequences whose n^{th} term is $a_n = n\frac{n^2+5}{4}$

Substituting n = 1, 2, 3, 4, 5, we obtain

$$a_1 = 1 \cdot \frac{1^2+5}{4} = \frac{6}{4} = \frac{3}{2}$$
$$a_2 = 2 \cdot \frac{2^2+5}{4} = 2 \cdot \frac{9}{4} = \frac{9}{2}$$
$$a_3 = 3 \cdot \frac{3^2+5}{4} = 3 \cdot \frac{14}{4} = \frac{21}{2}$$
$$a_4 = 4 \cdot \frac{4^2+5}{4} = 21$$
$$a_5 = 5 \cdot \frac{5^2+5}{4} = 5 \cdot \frac{30}{4} = \frac{75}{2}$$

Therefore, the required terms are $\frac{3}{2}, \frac{9}{2}, \frac{21}{2}$, 21, and $\frac{75}{2}$.

Question 7:

Find the 17th term in the following sequence whose n^{th} term is $a_n = 4n - 3; a_{17}, a_{24}$

Substituting n = 17, we obtain

$$a_{17} = 4(17) - 3 = 68 - 3 = 65$$

Substituting $n = 24$, we obtain

$$a_{24} = 4(24) - 3 = 96 - 3 = 93$$

Question 8:

Find the 7th term in the following sequence whose nth term is $a_n = \frac{n^2}{2n}; a_7$

Substituting $n = 7$, we obtain

$$a_7 = \frac{7^2}{2^7} = \frac{49}{128}$$

Question 9:

Find the 9th term in the following sequence whose nth term is $a_n = (-1)^{n-1} n^3; a_9$

Substituting $n = 9$, we obtain

$$a_9 = (-1)^{9-1}(9)^3 = (9)^3 = 729$$

Question 10:

Find the 20th term in the following sequence whose nth term is $a_n = \frac{n(n-2)}{n+3}; a_{20}$

Substituting $n = 20$, we obtain

$$a_{20} = \frac{20(20-2)}{20+3} = \frac{20(18)}{23} = \frac{360}{23}$$

Question 11:

Write the first five terms of the following sequence and obtain the corresponding series:

$$a_1 = 3, a_n = 3a_{n-1} + 2 \text{ for all } n > 1$$

$$a_1 = 3, a_n = 3a_{n-1} + 2 \text{ for all } n > 1$$

$$\Rightarrow a_2 = 3a_1 + 2 = 3(3) + 2 = 11$$
$$a_3 = 3a_2 + 2 = 3(11) + 2 = 35$$
$$a_4 = 3a_3 + 2 = 3(35) + 2 = 107$$
$$a_5 = 3a_4 + 2 = 3(107) + 2 = 323$$

Hence, the first five terms of the sequence are 3, 11, 35, 107, and 323.

The corresponding series is 3 + 11 + 35 + 107 + 323 + …

Question 12:

Write the first five terms of the following sequence and obtain the corresponding series:

$$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$$

$$a_1 = -1, a_n = \frac{a_{n-1}}{n}, n \geq 2$$

$$\Rightarrow a_2 = \frac{a_1}{2} = \frac{-1}{2}$$
$$a_3 = \frac{a_2}{3} = \frac{-1}{6}$$
$$a_4 = \frac{a_3}{4} = \frac{-1}{24}$$
$$a_5 = \frac{a_4}{4} = \frac{-1}{120}$$

Hence, the first five terms of the sequence are $-1, \frac{-1}{2}, \frac{-1}{6}, \frac{-1}{24}$, and $\frac{-1}{120}$.

The corresponding series is $(-1) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{6}\right) + \left(\frac{-1}{24}\right) + \left(\frac{-1}{120}\right) + \ldots$

Question 13:

Write the first five terms of the following sequence and obtain the corresponding series:

$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

$a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$

$$\begin{aligned} &\Rightarrow a_3 = a_2 - 1 = 2 - 1 = 1 \\ &a_4 = a_3 - 1 = 1 - 1 = 0 \\ &a_5 = a_4 - 1 = 0 - 1 = -1 \end{aligned}$$

Hence, the first five terms of the sequence are 2, 2, 1, 0, and –1.

The corresponding series is 2 + 2 + 1 + 0 + (–1) + …

Question 14:

The Fibonacci sequence is defined by

$1 = a_1 = a_2 \text{ and } a_n = a_{n-1} + a_{n-2}, n > 2$

Find $\frac{a_{n+1}}{a_n}$, for n = 1, 2, 3, 4, 5

$$\begin{aligned} &1 = a_1 = a_2 \\ &a_n = a_{n-1} + a_{n-2}, n > 2 \\ &\therefore a_3 = a_2 + a_1 = 1 + 1 = 2 \\ &a_4 = a_3 + a_2 = 2 + 1 = 3 \\ &a_5 = a_4 + a_3 = 3 + 2 = 5 \\ &a_6 = a_5 + a_4 = 5 + 3 = 8 \end{aligned}$$

$$\begin{aligned} &\therefore \text{For } n = 1, \ \frac{a_n + 1}{a_n} = \frac{a_2}{a_1} = \frac{1}{1} = 1 \\ &\text{For } n = 2, \ \frac{a_n + 1}{a_n} = \frac{a_3}{a_2} = \frac{2}{1} = 2 \\ &\text{For } n = 3, \frac{a_n + 1}{a_n} = \frac{a_4}{a_3} = \frac{3}{2} \\ &\text{For } n = 4, \ \frac{a_n + 1}{a_n} = \frac{a_5}{a_4} = \frac{5}{3} \\ &\text{For } n = 5, \ \frac{a_n + 1}{a_n} = \frac{a_6}{a_5} = \frac{8}{5} \end{aligned}$$

EXERCISE- 9.2

Question 1:

Find the sum of odd integers from 1 to 2001.

The odd integers from 1 to 2001 are 1, 3, 5, …1999, 2001.

This sequence forms an A.P.

Here, first term, $a = 1$

Common difference, $d = 2$

$$\begin{aligned}
&\text{Here, } a+(n-1)d = 2001 \\
&\Rightarrow 1+(n-1)(2) = 2001 \\
&\Rightarrow 2n-2 = 2000 \\
&\Rightarrow n = 1001
\end{aligned}$$

$$\begin{aligned}
S_n &= \frac{n}{2}\left[2a+(n-1)d\right] \\
\therefore S_n &= \frac{1001}{2}\left[2\times1+(1001-1)\times2\right] \\
&= \frac{1001}{2}\left[2+1000\times2\right] \\
&= \frac{1001}{2}\times2002 \\
&= 1001\times1001 \\
&= 1002001
\end{aligned}$$

Thus, the sum of odd numbers from 1 to 2001 is 1002001.

Question 2:

Find the sum of all natural numbers lying between 100 and 1000, which are multiples of 5.

The natural numbers lying between 100 and 1000, which are multiples of 5, are 105, 110, … 995.

Here, $a = 105$ and $d = 5$

$$a + (n-1)d = 995$$
$$\Rightarrow 105 + (n-1)5 = 995$$
$$\Rightarrow (n-1)5 = 995 - 105 = 890$$
$$\Rightarrow n - 1 = 178$$
$$\Rightarrow n = 179$$

$$\begin{aligned}\therefore S_n &= \frac{179}{2}\left[2(105) + (179-1)(5)\right] \\ &= \frac{179}{2}\left[2(105) + (178)(5)\right] \\ &= 179\left[105 + (89)5\right] \\ &= (179)(105 + 445) \\ &= (179)(550) \\ &= 98450\end{aligned}$$

Thus, the sum of all natural numbers lying between 100 and 1000, which are multiples of 5, is 98450.

Question 3:

In an A.P, the first term is 2 and the sum of the first five terms is one-fourth of the next five terms. Show that 20^{th} term is –112.

First term = 2

Let d be the common difference of the A.P.

Therefore, the A.P. is 2, $2 + d$, $2 + 2d$, $2 + 3d$, …

Sum of first five terms = $10 + 10d$

Sum of next five terms = $10 + 35d$

According to the given condition,

$$10+10d=\frac{1}{4}(10+35d)$$
$$\Rightarrow 40+40d=10+35d$$
$$\Rightarrow 30=-5d$$
$$\Rightarrow d=-6$$

$$\therefore a_{20}=a+(20-1)d=2+(19)(-6)=2-114=-112$$

Thus, the 20th term of the A.P. is –112.

Question 4:

How many terms of the A.P. $-6,-\frac{11}{2},-5,...$ are needed to give the sum –25?

Let the sum of n terms of the given A.P. be –25.

It is known that, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$, where n = number of terms, a = first term, and d = common difference

Here, a = –6

$$d=-\frac{11}{2}+6=\frac{-11+12}{2}=\frac{1}{2}$$

Therefore, we obtain

$$-25=\frac{n}{2}\left[2\times(-6)+(n-1)\left(\frac{1}{2}\right)\right]$$
$$\Rightarrow -50=n\left[-12+\frac{n}{2}-\frac{1}{2}\right]$$
$$\Rightarrow -50=n\left[-\frac{25}{2}+\frac{n}{2}\right]$$
$$\Rightarrow -100=n(-25+n)$$
$$\Rightarrow n^2-25n+100=0$$
$$\Rightarrow n^2-5n-20n+100=0$$
$$\Rightarrow n(n-5)-20(n-5)=0$$
$$\Rightarrow n=20 \text{ or } 5$$

Question 5:

In an A.P., if p^{th} term is $\frac{1}{q}$ and q^{th} term is $\frac{1}{p}$, prove that the sum of first pq terms is $\frac{1}{2}(pq+1)$ where $p \neq q$.

It is known that the general term of an A.P. is $a_n = a + (n-1)d$

$\therefore$ According to the given information,

$$p^{\text{th}} \text{ term} = a_p = a + (p-1)d = \frac{1}{q} \qquad \ldots(1)$$

$$q^{\text{th}} \text{ term} = a_q = a + (q-1)d = \frac{1}{p} \qquad \ldots(2)$$

Subtracting (2) from (1), we obtain

$$(p-1)d - (q-1)d = \frac{1}{q} - \frac{1}{p}$$

$$\Rightarrow (p-1-q+1)d = \frac{p-q}{pq}$$

$$\Rightarrow (p-q)d = \frac{p-q}{pq}$$

$$\Rightarrow d = \frac{1}{pq}$$

Putting the value of d in (1), we obtain

$$a + (p-1)\frac{1}{pq} = \frac{1}{q}$$

$$\Rightarrow a = \frac{1}{q} - \frac{1}{q} + \frac{1}{pq} = \frac{1}{pq}$$

$$\therefore S_{pq} = \frac{pq}{2}\left[2a + (pq-1)d\right]$$

$$= \frac{pq}{2}\left[\frac{2}{pq} + (pq-1)\frac{1}{pq}\right]$$

$$= 1 + \frac{1}{2}(pq-1)$$

$$= \frac{1}{2}pq + 1 - \frac{1}{2} = \frac{1}{2}pq + \frac{1}{2}$$

$$= \frac{1}{2}(pq+1)$$

Thus, the sum of first pq terms of the A.P. is $\frac{1}{2}(pq+1)$.

Question 6:

If the sum of a certain number of terms of the A.P. 25, 22, 19, … is 116. Find the last term

Let the sum of n terms of the given A.P. be 116.

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

Here, a = 25 and d = 22 – 25 = – 3

$$\therefore S_n = \frac{n}{2}\left[2\times 25+(n-1)(-3)\right]$$
$$\Rightarrow 116 = \frac{n}{2}[50-3n+3]$$
$$\Rightarrow 232 = n(53-3n) = 53n-3n^2$$
$$\Rightarrow 3n^2-53n+232=0$$
$$\Rightarrow 3n^2-24n-29n+232=0$$
$$\Rightarrow 3n(n-8)-29(n-8)=0$$
$$\Rightarrow (n-8)(3n-29)=0$$
$$\Rightarrow n=8 \text{ or } n=\frac{29}{3}$$

However, n cannot be equal to $\frac{29}{3}$. Therefore, n = 8

$\therefore a_8$ = Last term = $a + (n – 1)d$ = 25 + (8 – 1) (– 3)

= 25 + (7) (– 3) = 25 – 21

= 4

Thus, the last term of the A.P. is 4.

Question 7:

Find the sum to n terms of the A.P., whose k^{th} term is $5k + 1$.

It is given that the k^{th} term of the A.P. is $5k + 1$.

k^{th} term $= a_k = a + (k - 1)d$

$\therefore a + (k - 1)d = 5k + 1$

$a + kd - d = 5k + 1$

Comparing the coefficient of k, we obtain $d = 5$

$a - d = 1$

$\Rightarrow a - 5 = 1$

$\Rightarrow a = 6$

$$\begin{aligned} S_n &= \frac{n}{2}\left[2a + (n-1)d\right] \\ &= \frac{n}{2}\left[2(6) + (n-1)(5)\right] \\ &= \frac{n}{2}\left[12 + 5n - 5\right] \\ &= \frac{n}{2}(5n + 7) \end{aligned}$$

Question 8:

If the sum of n terms of an A.P. is $(pn + qn^2)$, where p and q are constants, find the common difference.

It is known that, $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

According to the given condition,

$$\frac{n}{2}\left[2a+(n-1)d\right]=pn+qn^2$$

$$\Rightarrow \frac{n}{2}\left[2a+nd-d\right]=pn+qn^2$$

$$\Rightarrow na+n^2\frac{d}{2}-n\cdot\frac{d}{2}=pn+qn^2$$

Comparing the coefficients of n^2 on both sides, we obtain

$$\frac{d}{2}=q$$

$\therefore d = 2q$

Thus, the common difference of the A.P. is $2q$.

Question 9:

The sums of n terms of two arithmetic progressions are in the ratio $5n + 4 : 9n + 6$. Find the ratio of their 18th terms.

Let a_1, a_2, and d_1, d_2 be the first terms and the common difference of the first and second arithmetic progression respectively.

According to the given condition,

$$\frac{\text{Sum of } n \text{ terms of first A.P.}}{\text{Sum of } n \text{ terms of second A.P.}}=\frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{\frac{n}{2}\left[2a_1+(n-1)d_1\right]}{\frac{n}{2}\left[2a_2+(n-1)d_2\right]}=\frac{5n+4}{9n+6}$$

$$\Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2}=\frac{5n+4}{9n+6} \qquad ...(1)$$

Substituting $n = 35$ in (1), we obtain

$$\frac{2a_1+34d_1}{2a_2+34d_2}=\frac{5(35)+4}{9(35)+6}$$

$$\Rightarrow \frac{a_1+17d_1}{a_2+17d_2}=\frac{179}{321} \qquad ...(2)$$

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P}} = \frac{a_1 + 17d_1}{a_2 + 17d_2} \qquad ...(3)$$

From (2) and (3), we obtain

$$\frac{18^{\text{th}} \text{ term of first A.P.}}{18^{\text{th}} \text{ term of second A.P.}} = \frac{179}{321}$$

Thus, the ratio of 18^{th} term of both the A.P.s is 179: 321.

Question 10:

If the sum of first *p* terms of an A.P. is equal to the sum of the first *q* terms, then find the sum of the first $(p + q)$ terms.

Let *a* and *d* be the first term and the common difference of the A.P. respectively.

Here,

$$S_p = \frac{p}{2}\left[2a + (p-1)d\right]$$
$$S_q = \frac{q}{2}\left[2a + (q-1)d\right]$$

According to the given condition,

$$\frac{p}{2}\left[2a+(p-1)d\right]=\frac{q}{2}\left[2a+(q-1)d\right]$$
$$\Rightarrow p\left[2a+(p-1)d\right]=q\left[2a+(q-1)d\right]$$
$$\Rightarrow 2ap+pd(p-1)=2aq+qd(q-1)$$
$$\Rightarrow 2a(p-q)+d\left[p(p-1)-q(q-1)\right]=0$$
$$\Rightarrow 2a(p-q)+d\left[p^2-p-q^2+q\right]=0$$
$$\Rightarrow 2a(p-q)+d\left[(p-q)(p+q)-(p-q)\right]=0$$
$$\Rightarrow 2a(p-q)+d\left[(p-q)(p+q-1)\right]=0$$
$$\Rightarrow 2a+d(p+q-1)=0$$
$$\Rightarrow d=\frac{-2a}{p+q-1} \qquad ...(1)$$
$$\therefore S_{p+q}=\frac{p+q}{2}\left[2a+(p+q-1)\cdot d\right]$$
$$\Rightarrow S_{p+q}=\frac{p+q}{2}\left[2a+(p+q-1)\left(\frac{-2a}{p+q-1}\right)\right] \qquad \text{[From (1)]}$$
$$=\frac{p+q}{2}\left[2a-2a\right]$$
$$=0$$

Thus, the sum of the first $(p + q)$ terms of the A.P. is 0.

Question 11:

Sum of the first p, q and r terms of an A.P. are a, b and c, respectively.

Prove that $\frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$

Let a_1 and d be the first term and the common difference of the A.P. respectively.

According to the given information,

$$S_p = \frac{p}{2}\left[2a_1 + (p-1)d\right] = a$$

$$\Rightarrow 2a_1 + (p-1)d = \frac{2a}{p} \qquad ...(1)$$

$$S_q = \frac{q}{2}\left[2a_1 + (q-1)d\right] = b$$

$$\Rightarrow 2a_1 + (q-1)d = \frac{2b}{q} \qquad ...(2)$$

$$S_r = \frac{r}{2}\left[2a_1 + (r-1)d\right] = c$$

$$\Rightarrow 2a_1 + (r-1)d = \frac{2c}{r} \qquad ...(3)$$

Subtracting (2) from (1), we obtain

$$(p-1)d - (q-1)d = \frac{2a}{p} - \frac{2b}{q}$$

$$\Rightarrow d(p-1-q+1) = \frac{2aq - 2bq}{pq}$$

$$\Rightarrow d(p-q) = \frac{2aq - 2bp}{pq}$$

$$\Rightarrow d = \frac{2(aq - bp)}{pq(p-q)} \qquad ...(4)$$

Subtracting (3) from (2), we obtain

$$(q-1)d - (r-1)d = \frac{2b}{q} - \frac{2c}{r}$$

$$\Rightarrow d(q-1-r+1) = \frac{2b}{q} - \frac{2c}{r}$$

$$\Rightarrow d(q-r) = \frac{2br - 2qc}{qr}$$

$$\Rightarrow d = \frac{2(br - qc)}{qr(q-r)} \qquad ...(5)$$

Equating both the values of d obtained in (4) and (5), we obtain

$$\frac{aq-bp}{pq(p-q)}=\frac{br-qc}{qr(q-r)}$$
$$\Rightarrow qr(q-r)(aq-bq)=pq(p-q)(br-qc)$$
$$\Rightarrow r(aq-bp)(q-r)=p(br-qc)(p-q)$$
$$\Rightarrow (aqr-bpr)(q-r)=(bpr-pqc)(p-q)$$

Dividing both sides by *pqr*, we obtain

$$\left(\frac{a}{p}-\frac{b}{q}\right)(q-r)=\left(\frac{b}{q}-\frac{c}{r}\right)(p-q)$$
$$\Rightarrow \frac{a}{p}(q-r)-\frac{b}{q}(q-r+p-q)+\frac{c}{r}(p-q)=0$$
$$\Rightarrow \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)=0$$

Thus, the given result is proved.

Question 12:

The ratio of the sums of m and n terms of an A.P. is m^2: n^2. Show that the ratio of m^{th} and n^{th} term is $(2m-1)$: $(2n-1)$.

Let a and b be the first term and the common difference of the A.P. respectively.

According to the given condition,

$$\frac{\text{Sum of m terms}}{\text{Sum of n terms}}=\frac{m^2}{n^2}$$
$$\Rightarrow \frac{\frac{m}{2}\left[2a+(m-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]}=\frac{m^2}{n^2}$$
$$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d}=\frac{m}{n} \qquad \ldots(1)$$

Putting $m = 2m - 1$ and $n = 2n - 1$ in (1), we obtain

$$\frac{2a+(2m-2)d}{2a+(2n-2)d}=\frac{2m-1}{2n-1}$$

$$\Rightarrow \frac{a+(m-1)d}{a+(n-1)d}=\frac{2m-1}{2n-1} \qquad ...(2)$$

$$\frac{m^{th}\text{ term of A.P.}}{n^{th}\text{ term of A.P.}}=\frac{a+(m-1)d}{a+(n-1)d} \qquad ...(3)$$

From (2) and (3), we obtain

$$\frac{m^{th}\text{ term of A.P}}{n^{th}\text{term of A.P}}=\frac{2m-1}{2n-1}$$

Thus, the given result is proved.

Question 13:

If the sum of n terms of an A.P. is $3n^2+5n$ and its m^{th} term is 164, find the value of m.

Let a and b be the first term and the common difference of the A.P. respectively.

$a_m = a + (m - 1)d = 164$ … (1)

Sum of n terms, $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

Here,

$$\frac{n}{2}\left[2a+nd-d\right]=3n^2+5n$$

$$\Rightarrow na+n^2\cdot\frac{d}{2}=3n^2+5n$$

Comparing the coefficient of n^2 on both sides, we obtain

$$\frac{d}{2}=3$$

$$\Rightarrow d=6$$

Comparing the coefficient of n on both sides, we obtain

$$a-\frac{d}{2}=5$$
$$\Rightarrow a-3=5$$
$$\Rightarrow a=8$$

Therefore, from (1), we obtain

$8 + (m - 1)\,6 = 164$

$\Rightarrow (m - 1)\,6 = 164 - 8 = 156$

$\Rightarrow m - 1 = 26$

$\Rightarrow m = 27$

Thus, the value of m is 27.

Question 14:

Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

Let A_1, A_2, A_3, A_4, and A_5 be five numbers between 8 and 26 such that

8, A_1, A_2, A_3, A_4, A_5, 26 is an A.P.

Here, $a = 8$, $b = 26$, $n = 7$

Therefore, $26 = 8 + (7 - 1)\,d$

$\Rightarrow 6d = 26 - 8 = 18$

$\Rightarrow d = 3$

$A_1 = a + d = 8 + 3 = 11$

$A_2 = a + 2d = 8 + 2 \times 3 = 8 + 6 = 14$

$A_3 = a + 3d = 8 + 3 \times 3 = 8 + 9 = 17$

$A_4 = a + 4d = 8 + 4 \times 3 = 8 + 12 = 20$

$A_5 = a + 5d = 8 + 5 \times 3 = 8 + 15 = 23$

Thus, the required five numbers between 8 and 26 are 11, 14, 17, 20, and 23.

Question 15:

If $\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$ is the A.M. between *a* and *b*, then find the value of *n*.

A.M. of *a* and *b* $=\frac{a+b}{2}$

According to the given condition,

$$\frac{a+b}{2}=\frac{a^n+b^n}{a^{n-1}+b^{n-1}}$$
$$\Rightarrow (a+b)\left(a^{n-1}+b^{n-1}\right)=2\left(a^n+b^n\right)$$
$$\Rightarrow a^n+ab^{n-1}+ba^{n-1}+b^n=2a^n+2b^n$$
$$\Rightarrow ab^{n-1}+a^{n-1}b=a^n+b^n$$
$$\Rightarrow ab^{n-1}-b^n=a^n-a^{n-1}b$$
$$\Rightarrow b^{n-1}(a-b)=a^{n-1}(a-b)$$
$$\Rightarrow b^{n-1}=a^{n-1}$$
$$\Rightarrow \left(\frac{a}{b}\right)^{n-1}=1=\left(\frac{a}{b}\right)^0$$
$$\Rightarrow n-1=0$$
$$\Rightarrow n=1$$

Question 16:

Between 1 and 31, *m* numbers have been inserted in such a way that the resulting sequence is an A.P. and the ratio of 7th and $(m-1)$th numbers is 5:9. Find the value of *m*.

Let $A_1, A_2, \ldots A_m$ be *m* numbers such that $1, A_1, A_2, \ldots A_m, 31$ is an A.P.

Here, $a=1$, $b=31$, $n=m+2$

$\therefore 31=1+(m+2-1)(d)$

$\Rightarrow 30=(m+1)d$

$\Rightarrow d = \dfrac{30}{m+1}$ ...(1)

$A_1 = a + d$

$A_2 = a + 2d$

$A_3 = a + 3d$ …

$\therefore A_7 = a + 7d$

$A_{m-1} = a + (m - 1)\, d$

According to the given condition,

$$\frac{a+7d}{a+(m-1)d} = \frac{5}{9}$$

$$\Rightarrow \frac{1+7\left(\frac{30}{(m+1)}\right)}{1+(m-1)\left(\frac{30}{m+1}\right)} = \frac{5}{9} \qquad \text{[From (1)]}$$

$$\Rightarrow \frac{m+1+7(30)}{m+1+30(m-1)} = \frac{5}{9}$$

$$\Rightarrow \frac{m+1+210}{m+1+30m-30} = \frac{5}{9}$$

$$\Rightarrow \frac{m+211}{31m-29} = \frac{5}{9}$$

$$\Rightarrow 9m + 1899 = 155m - 145$$

$$\Rightarrow 155m - 9m = 1899 + 145$$

$$\Rightarrow 146m = 2044$$

$$\Rightarrow m = 14$$

Thus, the value of m is 14.

Question 17:

A man starts repaying a loan as first installment of Rs. 100. If he increases the installment by Rs 5 every month, what amount he will pay in the 30th installment?

The first installment of the loan is Rs 100.

The second installment of the loan is Rs 105 and so on.

The amount that the man repays every month forms an A.P.

The A.P. is 100, 105, 110, …

First term, $a = 100$

Common difference, $d = 5$

$A_{30} = a + (30 - 1)d$

$= 100 + (29)(5)$

$= 100 + 145$

$= 245$

Thus, the amount to be paid in the 30th installment is Rs 245.

Question 18:

The difference between any two consecutive interior angles of a polygon is 5°. If the smallest angle is 120°, find the number of the sides of the polygon.

The angles of the polygon will form an A.P. with common difference d as 5° and first term a as 120°.

It is known that the sum of all angles of a polygon with n sides is $180° (n - 2)$.

$\therefore S_n = 180°(n-2)$

$\Rightarrow \frac{n}{2}\left[2a+(n-1)d\right] = 180°(n-2)$

$\Rightarrow \frac{n}{2}\left[240°+(n-1)5°\right] = 180(n-2)$

$\Rightarrow n\left[240+(n-1)5\right] = 360(n-2)$

$\Rightarrow 240n+5n^2-5n = 360n-720$

$\Rightarrow 5n^2+235n-360n+720 = 0$

$\Rightarrow 5n^2-125n+720 = 0$

$\Rightarrow n^2-25n+144 = 0$

$\Rightarrow n^2-16n-9n+144 = 0$

$\Rightarrow n(n-16)-9(n-16) = 0$

$\Rightarrow (n-9)(n-16) = 0$

$\Rightarrow n = 9 \text{ or } 16$

EXERCISE- 9.3

Question 1:

Find the 20th and nth terms of the G.P. $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

The given G.P. is $\frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots$

Here, a = First term = $\frac{5}{2}$

r = Common ratio = $\frac{\frac{5}{4}}{\frac{5}{2}} = \frac{1}{2}$

$$a_{20} = ar^{20-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{19} = \frac{5}{(2)(2)^{19}} = \frac{5}{(2)^{20}}$$

$$a_n = ar^{n-1} = \frac{5}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{5}{(2)(2)^{n-1}} = \frac{5}{(2)^n}$$

Question 2:

Find the 12th term of a G.P. whose 8th term is 192 and the common ratio is 2.

Common ratio, $r = 2$

Let a be the first term of the G.P.

$\therefore a_8 = ar^{8-1} = ar^7$

$\Rightarrow ar^7 = 192$

$a(2)^7 = 192$

$a(2)^7 = (2)^6\,(3)$

$$\Rightarrow a = \frac{(2)^6 \times 3}{(2)^7} = \frac{3}{2}$$

$$\therefore a_{12} = ar^{12-1} = \left(\frac{3}{2}\right)(2)^{11} = (3)(2)^{10} = 3072$$

Question 3:

The 5th, 8th and 11th terms of a G.P. are p, q and s, respectively. Show that $q^2 = ps$.

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_5 = a\,r^{5-1} = a\,r^4 = p$ … (1)

$a_8 = a\,r^{8-1} = a\,r^7 = q$ … (2)

$a_{11} = a\,r^{11-1} = a\,r^{10} = s$ … (3)

Dividing equation (2) by (1), we obtain

$$\frac{a r^7}{a r^4} = \frac{q}{p}$$

$$r^3 = \frac{q}{p} \qquad \text{...(4)}$$

Dividing equation (3) by (2), we obtain

$$\frac{a r^{10}}{a r^7} = \frac{s}{q}$$

$$\Rightarrow r^3 = \frac{s}{q} \qquad \text{...(5)}$$

Equating the values of r^3 obtained in (4) and (5), we obtain

$$\frac{q}{p} = \frac{s}{q}$$

$$\Rightarrow q^2 = ps$$

Thus, the given result is proved.

Question 4:

The 4th term of a G.P. is square of its second term, and the first term is –3. Determine its 7th term.

Let a be the first term and r be the common ratio of the G.P.

$\therefore a = -3$

It is known that, $a_n = ar^{n-1}$

$\therefore a_4 = ar^3 = (-3)\, r^3$

$a_2 = a\,r^1 = (-3)\,r$

According to the given condition,

$(-3)\,r^3 = [\,(-3)\,r]^2$

$\Rightarrow -3r^3 = 9\,r^2$

$\Rightarrow r = -3$

$a_7 = a\,r^{7-1} = a\,r^6 = (-3)\,(-3)^6 = -\,(3)^7 = -2187$

Thus, the seventh term of the G.P. is –2187.

Question 5:

Which term of the following sequences:

(a) $2, 2\sqrt{2}, 4, \ldots$ is 128? (b) $\sqrt{3}, 3, 3\sqrt{3}, \ldots$ is 729? (c) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$ is $\frac{1}{19683}$?

(a) The given sequence is $2, 2\sqrt{2}, 4, \ldots$

Here, $a = 2$ and $r = \frac{2\sqrt{2}}{2} = \sqrt{2}$

Let the n^{th} term of the given sequence be 128.

$$\begin{aligned}
& a_n = a\,r^{n-1} \\
& \Rightarrow (2)\left(\sqrt{2}\right)^{n-1} = 128 \\
& \Rightarrow (2)(2)^{\frac{n-1}{2}} = (2)^7 \\
& \Rightarrow (2)^{\frac{n-1}{2}+1} = (2)^7 \\
& \therefore \frac{n-1}{2} + 1 = 7 \\
& \Rightarrow \frac{n-1}{2} = 6 \\
& \Rightarrow n - 1 = 12 \\
& \Rightarrow n = 13
\end{aligned}$$

Thus, the 13^{th} term of the given sequence is 128.

(b) The given sequence is $\sqrt{3}, 3, 3\sqrt{3},...$

Here, $a=\sqrt{3}$ and $r=\frac{3}{\sqrt{3}}=\sqrt{3}$

Let the n^{th} term of the given sequence be 729.

$$a_n = ar^{n-1}$$
$$\therefore ar^{n-1} = 729$$
$$\Rightarrow \left(\sqrt{3}\right)\left(\sqrt{3}\right)^{n-1} = 729$$
$$\Rightarrow (3)^{\frac{1}{2}}(3)^{\frac{n-1}{2}} = (3)^6$$
$$\Rightarrow (3)^{\frac{1}{2}+\frac{n-1}{2}} = (3)^6$$

$$\therefore \frac{1}{2}+\frac{n-1}{2} = 6$$
$$\Rightarrow \frac{1+n-1}{2} = 6$$
$$\Rightarrow n = 12$$

Thus, the 12^{th} term of the given sequence is 729.

(c) The given sequence is $\frac{1}{3},\frac{1}{9},\frac{1}{27},...$

Here, $a=\frac{1}{3}$ and $r=\frac{1}{9}\div\frac{1}{3}=\frac{1}{3}$

Let the n^{th} term of the given sequence be $\frac{1}{19683}$.

$a_n = ar^{n-1}$

$\therefore ar^{n-1} = \dfrac{1}{19683}$

$\Rightarrow \left(\dfrac{1}{3}\right)\left(\dfrac{1}{3}\right)^{n-1} = \dfrac{1}{19683}$

$\Rightarrow \left(\dfrac{1}{3}\right)^{n} = \left(\dfrac{1}{3}\right)^{9}$

$\Rightarrow n = 9$

Thus, the 9th term of the given sequence is $\dfrac{1}{19683}$.

Question 6:

For what values of *x*, the numbers $\dfrac{2}{7}, x, -\dfrac{7}{2}$ are in G.P?

The given numbers are $\dfrac{-2}{7}, x, \dfrac{-7}{2}$.

Common ratio = $\dfrac{x}{\frac{-2}{7}} = \dfrac{-7x}{2}$

Also, common ratio = $\dfrac{\frac{-7}{2}}{x} = \dfrac{-7}{2x}$

$\therefore \dfrac{-7x}{2} = \dfrac{-7}{2x}$

$\Rightarrow x^2 = \dfrac{-2\times 7}{-2\times 7} = 1$

$\Rightarrow x = \sqrt{1}$

$\Rightarrow x = \pm 1$

Thus, for $x = \pm 1$, the given numbers will be in G.P.

Question 7:

Find the sum to 20 terms in the geometric progression 0.15, 0.015, 0.0015 …

The given G.P. is 0.15, 0.015, 0.00015, …

Here, a = 0.15 and $r = \frac{0.015}{0.15} = 0.1$

$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$
$$\therefore S_{20} = \frac{0.15\left[1-(0.1)^{20}\right]}{1-0.1}$$
$$= \frac{0.15}{0.9}\left[1-(0.1)^{20}\right]$$
$$= \frac{15}{90}\left[1-(0.1)^{20}\right]$$
$$= \frac{1}{6}\left[1-(0.1)^{20}\right]$$

Question 8:

Find the sum to n terms in the geometric progression $\sqrt{7}, \sqrt{21}, 3\sqrt{7}...$

The given G.P. is $\sqrt{7}, \sqrt{21}, 3\sqrt{7},...$

Here, $a = \sqrt{7}$

$$r = \frac{\sqrt{21}}{\sqrt{7}} = \sqrt{3}$$
$$S_n = \frac{a\left(1-r^n\right)}{1-r}$$
$$\therefore S_n = \frac{\sqrt{7}\left[1-\left(\sqrt{3}\right)^n\right]}{1-\sqrt{3}}$$
$$= \frac{\sqrt{7}\left[1-\left(\sqrt{3}\right)^n\right]}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} \qquad \text{(By rationalizing)}$$
$$= \frac{\sqrt{7}\left(1+\sqrt{3}\right)\left[1-\left(\sqrt{3}\right)^n\right]}{1-3}$$
$$= \frac{-\sqrt{7}\left(1+\sqrt{3}\right)}{2}\left[1-(3)^{\frac{n}{2}}\right]$$
$$= \frac{\sqrt{7}\left(1+\sqrt{3}\right)}{2}\left[(3)^{\frac{n}{2}}-1\right]$$

Question 9:

Find the sum to n terms in the geometric progression $1, -a, a^2, -a^3 \ldots \left(\text{if } a \neq -1\right)$

The given G.P. is $1, -a, a^2, -a^3, \ldots\ldots\ldots\ldots$

Here, first term = a_1 = 1

Common ratio = r = – a

$$S_n = \frac{a_1\left(1-r^n\right)}{1-r}$$
$$\therefore S_n = \frac{1\left[1-(-a)^n\right]}{1-(-a)} = \frac{\left[1-(-a)^n\right]}{1+a}$$

Question 10:

Find the sum to n terms in the geometric progression $x^3, x^5, x^7 \ldots \left(\text{if } x \neq \pm 1\right)$

The given G.P. is $x^3, x^5, x^7, \ldots$

Here, $a = x^3$ and $r = x^2$

$$S_n = \frac{a\left(1-r^n\right)}{1-r} = \frac{x^3\left[1-\left(x^2\right)^n\right]}{1-x^2} = \frac{x^3\left(1-x^{2n}\right)}{1-x^2}$$

Question 11:

Evaluate $\sum_{k=1}^{11}\left(2+3^k\right)$

$$\sum_{k=1}^{11}\left(2+3^k\right) = \sum_{k=1}^{11}(2) + \sum_{k=1}^{11}3^k = 2(11) + \sum_{k=1}^{11}3^k = 22 + \sum_{k=1}^{11}3^k \qquad \ldots(1)$$
$$\sum_{k=1}^{11}3^k = 3^1 + 3^2 + 3^3 + \ldots + 3^{11}$$

The terms of this sequence 3, 3^2, 3^3, … forms a G.P.

$$S_n = \frac{a\left(r^n - 1\right)}{r-1}$$
$$\Rightarrow S_{11} = \frac{3\left[(3)^{11} - 1\right]}{3-1}$$
$$\Rightarrow S_{11} = \frac{3}{2}\left(3^{11} - 1\right)$$
$$\therefore \sum_{k=1}^{11} 3^k = \frac{3}{2}\left(3^{11} - 1\right)$$

Substituting this value in equation (1), we obtain

$$\sum_{k=1}^{11}\left(2 + 3^k\right) = 22 + \frac{3}{2}\left(3^{11} - 1\right)$$

Question 12:

The sum of first three terms of a G.P. is $\frac{39}{10}$ and their product is 1. Find the common ratio and the terms.

Let $\frac{a}{r}, a, ar$ be the first three terms of the G.P.

$$\frac{a}{r} + a + ar = \frac{39}{10} \qquad \ldots(1)$$
$$\left(\frac{a}{r}\right)(a)(ar) = 1 \qquad \ldots(2)$$

From (2), we obtain

$a^3 = 1$

$\Rightarrow a = 1$ (Considering real roots only)

Substituting $a = 1$ in equation (1), we obtain

$$\frac{1}{r}+1+r=\frac{39}{10}$$
$$\Rightarrow 1+r+r^2=\frac{39}{10}r$$
$$\Rightarrow 10+10r+10r^2-39r=0$$
$$\Rightarrow 10r^2-29r+10=0$$
$$\Rightarrow 10r^2-25r-4r+10=0$$
$$\Rightarrow 5r(2r-5)-2(2r-5)=0$$
$$\Rightarrow (5r-2)(2r-5)=0$$
$$\Rightarrow r=\frac{2}{5} \text{ or } \frac{5}{2}$$

Thus, the three terms of G.P. are $\frac{5}{2}, 1, \text{and } \frac{2}{5}$.

Question 13:

How many terms of G.P. $3, 3^2, 3^3, \ldots$ are needed to give the sum 120?

The given G.P. is $3, 3^2, 3^3, \ldots$

Let *n* terms of this G.P. be required to obtain the sum as 120.

$$S_n=\frac{a\left(r^n-1\right)}{r-1}$$

Here, $a = 3$ and $r = 3$

$$\therefore S_n = 120 = \frac{3(3^n - 1)}{3 - 1}$$

$$\Rightarrow 120 = \frac{3(3^n - 1)}{2}$$

$$\Rightarrow \frac{120 \times 2}{3} = 3^n - 1$$

$$\Rightarrow 3^n - 1 = 80$$

$$\Rightarrow 3^n = 81$$

$$\Rightarrow 3^n = 3^4$$

$\therefore n = 4$

Thus, four terms of the given G.P. are required to obtain the sum as 120.

Question 14:

The sum of first three terms of a G.P. is 16 and the sum of the next three terms is 128. Determine the first term, the common ratio and the sum to n terms of the G.P.

Let the G.P. be a, ar, ar^2, ar^3, …

According to the given condition,

$a + ar + ar^2 = 16$ and $ar^3 + ar^4 + ar^5 = 128$

$\Rightarrow a(1 + r + r^2) = 16$ … (1)

$ar^3(1 + r + r^2) = 128$ … (2)

Dividing equation (2) by (1), we obtain

$$\frac{ar^3(1 + r + r^2)}{a(1 + r + r^2)} = \frac{128}{16}$$

$$\Rightarrow r^3 = 8$$

$$\therefore r = 2$$

Substituting $r = 2$ in (1), we obtain

$a(1 + 2 + 4) = 16$

$\Rightarrow a\,(7) = 16$

$$\Rightarrow a = \frac{16}{7}$$

$$S_n = \frac{a\left(r^n - 1\right)}{r - 1}$$

$$\Rightarrow S_n = \frac{16}{7}\frac{\left(2^n - 1\right)}{2 - 1} = \frac{16}{7}\left(2^n - 1\right)$$

Question 15:

Given a G.P. with $a = 729$ and 7^{th} term 64, determine S_7.

$a = 729$

$a_7 = 64$

Let r be the common ratio of the G.P.

It is known that, $a_n = a\,r^{n-1}$

$a_7 = ar^{7-1} = (729)r^6$

$\Rightarrow 64 = 729\,r^6$

$$\Rightarrow r^6 = \frac{64}{729}$$

$$\Rightarrow r^6 = \left(\frac{2}{3}\right)^6$$

$$\Rightarrow r = \frac{2}{3}$$

Also, it is known that, $S_n = \dfrac{a\left(1 - r^n\right)}{1 - r}$

$$\therefore S_7 = \frac{729\left[1-\left(\frac{2}{3}\right)^7\right]}{1-\frac{2}{3}}$$

$$= 3 \times 729\left[1-\left(\frac{2}{3}\right)^7\right]$$

$$= (3)^7\left[\frac{(3)^7-(2)^7}{(3)^7}\right]$$

$$= (3)^7-(2)^7$$

$$= 2187-128$$

$$= 2059$$

Question 16:

Find a G.P. for which sum of the first two terms is –4 and the fifth term is 4 times the third term.

Let a be the first term and r be the common ratio of the G.P.

According to the given conditions,

$$S_2 = -4 = \frac{a\left(1-r^2\right)}{1-r} \qquad \ldots(1)$$

$a_5 = 4 \times a_3$

$ar^4 = 4ar^2$

$\Rightarrow r^2 = 4$

$\therefore r = \pm 2$

From (1), we obtain

$-4=\frac{a\left[1-(2)^2\right]}{1-2}$ for $r=2$

$\Rightarrow -4=\frac{a(1-4)}{-1}$

$\Rightarrow -4=a(3)$

$\Rightarrow a=\frac{-4}{3}$

Also, $-4=\frac{a\left[1-(-2)^2\right]}{1-(-2)}$ for $r=-2$

$\Rightarrow -4=\frac{a(1-4)}{1+2}$

$\Rightarrow -4=\frac{a(-3)}{3}$

$\Rightarrow a=4$

Thus, the required G.P. is

$\frac{-4}{3},\frac{-8}{3},\frac{-16}{3},\ldots$ or 4, –8, 16, –32, …

Question 17:

If the 4th, 10th and 16th terms of a G.P. are x, y and z, respectively. Prove that x, y, z are in G.P.

Let a be the first term and r be the common ratio of the G.P.

According to the given condition,

$a_4 = a r^3 = x$ … (1)

$a_{10} = a r^9 = y$ … (2)

$a_{16} = a r^{15} = z$ … (3)

Dividing (2) by (1), we obtain

$\frac{y}{x}=\frac{ar^9}{ar^3}\Rightarrow \frac{y}{x}=r^6$

Dividing (3) by (2), we obtain

$$\frac{z}{y}=\frac{ar^{15}}{ar^{9}}\Rightarrow\frac{z}{y}=r^{6}$$

$$\therefore\ \frac{y}{x}=\frac{z}{y}$$

Thus, x, y, z are in G. P.

Question 18:

Find the sum to n terms of the sequence, 8, 88, 888, 8888…

The given sequence is 8, 88, 888, 8888…

This sequence is not a G.P. However, it can be changed to G.P. by writing the terms as

S_n = 8 + 88 + 888 + 8888 + …………….. to n terms

$$=\frac{8}{9}[9+99+999+9999+\text{..........to } n \text{ terms}]$$
$$=\frac{8}{9}\left[(10-1)+(10^2-1)+(10^3-1)+(10^4-1)+\text{........to } n \text{ terms}\right]$$
$$=\frac{8}{9}\left[(10+10^2+\text{......}n\text{ terms})-(1+1+1+\text{....}n\text{ terms})\right]$$
$$=\frac{8}{9}\left[\frac{10(10^n-1)}{10-1}-n\right]$$
$$=\frac{8}{9}\left[\frac{10(10^n-1)}{9}-n\right]$$
$$=\frac{80}{81}(10^n-1)-\frac{8}{9}n$$

Question 19:

Find the sum of the products of the corresponding terms of the sequences 2, 4, 8, 16, 32 and 128, 32, 8, 2, $\frac{1}{2}$.

Required sum = $2\times128+4\times32+8\times8+16\times2+32\times\frac{1}{2}$

$$=64\left[4+2+1+\frac{1}{2}+\frac{1}{2^2}\right]$$

Here, 4, 2, 1, $\frac{1}{2},\frac{1}{2^2}$ is a G.P.

First term, $a = 4$

Common ratio, $r = \frac{1}{2}$

It is known that, $S_n=\frac{a\left(1-r^n\right)}{1-r}$

$$\therefore S_5=\frac{4\left[1-\left(\frac{1}{2}\right)^5\right]}{1-\frac{1}{2}}=\frac{4\left[1-\frac{1}{32}\right]}{\frac{1}{2}}=8\left(\frac{32-1}{32}\right)=\frac{31}{4}$$

$\therefore$Required sum = $64\left(\frac{31}{4}\right)=(16)(31)=496$

Question 20:

Show that the products of the corresponding terms of the sequences $a, ar, ar^2, \ldots ar^{n-1}$ and $A, AR, AR^2, \ldots AR^{n-1}$ form a G.P, and find the common ratio.

It has to be proved that the sequence, aA, $arAR$, ar^2AR^2, $\ldots ar^{n-1}AR^{n-1}$, forms a G.P.

$$\frac{\text{Second term}}{\text{First term}}=\frac{arAR}{aA}=rR$$
$$\frac{\text{Third term}}{\text{Second term}}=\frac{ar^2AR^2}{arAR}=rR$$

Thus, the above sequence forms a G.P. and the common ratio is rR.

Question 21:

Find four numbers forming a geometric progression in which third term is greater than the first term by 9, and the second term is greater than the 4th by 18.

Let a be the first term and r be the common ratio of the G.P.

$a_1 = a$, $a_2 = ar$, $a_3 = ar^2$, $a_4 = ar^3$

By the given condition,

$a_3 = a_1 + 9$

$\Rightarrow ar^2 = a + 9 \ldots (1)$

$a_2 = a_4 + 18$

$\Rightarrow ar = ar^3 + 18 \ldots (2)$

From (1) and (2), we obtain

$a(r^2 - 1) = 9 \ldots (3)$

$ar\,(1 - r^2) = 18 \ldots (4)$

Dividing (4) by (3), we obtain

$$\frac{ar\left(1-r^2\right)}{a\left(r^2-1\right)} = \frac{18}{9}$$
$$\Rightarrow -r = 2$$
$$\Rightarrow r = -2$$

Substituting the value of r in (1), we obtain

$4a = a + 9$

$\Rightarrow 3a = 9$

$\therefore a = 3$

Thus, the first four numbers of the G.P. are 3, 3(– 2), $3(-2)^2$, and $3(-2)^3$ i.e., 3,–6, 12, and –24.

Question 22:

If the p^{th}, q^{th} and r^{th} terms of a G.P. are *a, b* and *c*, respectively. Prove that $a^{q-r} b^{r-p} c^{p-q} = 1$

Let *A* be the first term and *R* be the common ratio of the G.P.

According to the given information,

$AR^{p-1} = a$

$AR^{q-1} = b$

$AR^{r-1} = c$

$a^{q-r} b^{r-p} c^{p-q}$

$= A^{q-r} \times R^{(p-1)(q-r)} \times A^{r-p} \times R^{(q-1)(r-p)} \times A^{p-q} \times R^{(r-1)(p-q)}$

$= Aq^{-r+r-p+p-q} \times R^{(pr-pr-q+r)+(rq-r+p-pq)+(pr-p-qr+q)}$

$= A^0 \times R^0$

$= 1$

Thus, the given result is proved.

Question 23:

If the first and the n^{th} term of a G.P. are *a* ad *b*, respectively, and if *P* is the product of *n* terms, prove that $P^2 = (ab)^n$.

The first term of the G.P is *a* and the last term is *b*.

Therefore, the G.P. is $a, ar, ar^2, ar^3, \ldots ar^{n-1}$, where *r* is the common ratio.

$b = ar^{n-1} \ldots (1)$

P = Product of n terms

$= (a)(ar)(ar^2) \ldots (ar^{n-1})$

$= (a \times a \times \ldots a)(r \times r^2 \times \ldots r^{n-1})$

$= a^n r^{1+2+\ldots(n-1)} \ldots (2)$

Here, 1, 2, …(n – 1) is an A.P.

$\therefore 1 + 2 + \ldots\ldots\ldots + (n-1) = \frac{n-1}{2}\left[2+(n-1-1)\times 1\right] = \frac{n-1}{2}\left[2+n-2\right] = \frac{n(n-1)}{2}$

$$P = a^n r^{\frac{n(n-1)}{2}}$$
$$\therefore P^2 = a^{2n} r^{n(n-1)}$$
$$= \left[a^2 r^{(n-1)}\right]^n$$
$$= \left[a \times ar^{n-1}\right]^n$$
$$= (ab)^n \qquad \left[\text{Using } (1)\right]$$

Thus, the given result is proved.

Question 24:

Show that the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$ term is $\frac{1}{r^n}$.

Let a be the first term and r be the common ratio of the G.P.

$$\text{Sum of first n terms} = \frac{a(1-r^n)}{(1-r)}$$

Since there are n terms from $(n+1)^{th}$ to $(2n)^{th}$ term, Sum of terms from$(n+1)^{th}$ to $(2n)^{th}$ term $=\dfrac{a_{n+1}\left(1-r^n\right)}{(1-r)}$

$a^{n+1} = ar^{n+1-1} = ar^n$

Thus, required ratio = $\dfrac{a\left(1-r^n\right)}{(1-r)}\times\dfrac{(1-r)}{ar^n\left(1-r^n\right)}=\dfrac{1}{r^n}$

Thus, the ratio of the sum of first n terms of a G.P. to the sum of terms from $(n+1)^{th}$ to $(2n)^{th}$term is $\dfrac{1}{r^n}$.

Question 25:

If a, b, c and d are in G.P. show that $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=\left(ab+bc+cd\right)^2$.

a, b, c, d are in G.P.

Therefore,

$bc = ad$ … (1)

$b^2 = ac$ … (2)

$c^2 = bd$ … (3)

It has to be proved that,

$(a^2 + b^2 + c^2)\ (b^2 + c^2 + d^2) = (ab + bc - cd)^2$

R.H.S.

$= (ab + bc + cd)^2$

$= (ab + ad + cd)^2$ [Using (1)]

$= [ab + d\ (a + c)]^2$

$= a^2b^2 + 2abd\ (a + c) + d^2\ (a + c)^2$

$= a^2b^2 + 2a^2bd + 2acbd + d^2(a^2 + 2ac + c^2)$

$= a^2b^2 + 2a^2c^2 + 2b^2c^2 + d^2a^2 + 2d^2b^2 + d^2c^2$ [Using (1) and (2)]

$= a^2b^2 + a^2c^2 + a^2c^2 + b^2c^2 + b^2c^2 + d^2a^2 + d^2b^2 + d^2b^2 + d^2c^2$

$= a^2b^2 + a^2c^2 + a^2d^2 + b^2 \times b^2 + b^2c^2 + b^2d^2 + c^2b^2 + c^2 \times c^2 + c^2d^2$

[Using (2) and (3) and rearranging terms]

$= a^2(b^2 + c^2 + d^2) + b^2 (b^2 + c^2 + d^2) + c^2 (b^2 + c^2 + d^2)$

$= (a^2 + b^2 + c^2) (b^2 + c^2 + d^2)$

= L.H.S.

∴ L.H.S. = R.H.S.

∴ $\left(a^2+b^2+c^2\right)\left(b^2+c^2+d^2\right)=\left(ab+bc+cd\right)^2$

Question 26:

Insert two numbers between 3 and 81 so that the resulting sequence is G.P.

Let G_1 and G_2 be two numbers between 3 and 81 such that the series, 3, G_1, G_2, 81, forms a G.P.

Let a be the first term and r be the common ratio of the G.P.

$\therefore 81 = (3)\ (r)^3$

$\Rightarrow r^3 = 27$

$\therefore r = 3$ (Taking real roots only)

For $r = 3$,

$G_1 = ar = (3)\ (3) = 9$

$G_2 = ar^2 = (3)(3)^2 = 27$

Thus, the required two numbers are 9 and 27.

Question 27:

Find the value of n so that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ may be the geometric mean between a and b.

G. M. of a and b is $\sqrt{ab}$.

By the given condition, $\frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \sqrt{ab}$

Squaring both sides, we obtain

$$\frac{\left(a^{n+1}+b^{n+1}\right)^2}{\left(a^n+b^n\right)^2} = ab$$
$$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2} = (ab)\left(a^{2n}+2a^nb^n+b^{2n}\right)$$
$$\Rightarrow a^{2n+2}+2a^{n+1}b^{n+1}+b^{2n+2} = a^{2n+1}b+2a^{n+1}b^{n+1}+ab^{2n+1}$$
$$\Rightarrow a^{2n+2}+b^{2n+2} = a^{2n+1}b+ab^{2n+1}$$
$$\Rightarrow a^{2n+2}-a^{2n+1}b = ab^{2n+1}-b^{2n+2}$$
$$\Rightarrow a^{2n+1}(a-b) = b^{2n+1}(a-b)$$
$$\Rightarrow \left(\frac{a}{b}\right)^{2n+1} = 1 = \left(\frac{a}{b}\right)^0$$
$$\Rightarrow 2n+1 = 0$$
$$\Rightarrow n = \frac{-1}{2}$$

Question 28:

The sum of two numbers is 6 times their geometric mean, show that numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Let the two numbers be a and b.

G.M. = $\sqrt{ab}$

According to the given condition,

$$a+b=6\sqrt{ab} \qquad ...(1)$$
$$\Rightarrow (a+b)^2 = 36(ab)$$

Also,

$$(a-b)^2=(a+b)^2-4ab=36ab-4ab=32ab$$
$$\Rightarrow a-b=\sqrt{32}\sqrt{ab}$$
$$=4\sqrt{2}\sqrt{ab} \qquad ...(2)$$

Adding (1) and (2), we obtain

$$2a=\left(6+4\sqrt{2}\right)\sqrt{ab}$$
$$\Rightarrow a=\left(3+2\sqrt{2}\right)\sqrt{ab}$$

Substituting the value of *a* in (1), we obtain

$$b=6\sqrt{ab}-\left(3+2\sqrt{2}\right)\sqrt{ab}$$
$$\Rightarrow b=\left(3-2\sqrt{2}\right)\sqrt{ab}$$
$$\frac{a}{b}=\frac{\left(3+2\sqrt{2}\right)\sqrt{ab}}{\left(3-2\sqrt{2}\right)\sqrt{ab}}=\frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

Thus, the required ratio is $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$.

Question 29:

If A and G be A.M. and G.M., respectively between two positive numbers, prove that the numbers are $A \pm \sqrt{(A+G)(A-G)}$.

It is given that A and G are A.M. and G.M. between two positive numbers. Let these two positive numbers be a and b.

$$\therefore \text{AM} = A = \frac{a+b}{2} \qquad ...(1)$$

$$\text{GM} = G = \sqrt{ab} \qquad ...(2)$$

From (1) and (2), we obtain

$a + b = 2A$ … (3)

$ab = G^2$ … (4)

Substituting the value of a and b from (3) and (4) in the identity $(a - b)^2 = (a + b)^2 - 4ab$, we obtain

$(a - b)^2 = 4A^2 - 4G^2 = 4\ (A^2 - G^2)$

$(a - b)^2 = 4\ (A + G)\ (A - G)$

$$(a-b) = 2\sqrt{(A+G)(A-G)} \qquad ...(5)$$

From (3) and (5), we obtain

$$2a = 2A + 2\sqrt{(A+G)(A-G)}$$

$$\Rightarrow a = A + \sqrt{(A+G)(A-G)}$$

Substituting the value of a in (3), we obtain

$$b = 2A - A - \sqrt{(A+G)(A-G)} = A - \sqrt{(A+G)(A-G)}$$

Thus, the two numbers are $A \pm \sqrt{(A+G)(A-G)}$.

Question 30:

The number of bacteria in a certain culture doubles every hour. If there were 30 bacteria present in the culture originally, how many bacteria will be present at the end of 2nd hour, 4th hour and nth hour?

It is given that the number of bacteria doubles every hour. Therefore, the number of bacteria after every hour will form a G.P.

Here, $a = 30$ and $r = 2$

$\therefore a_3 = ar^2 = (30)(2)^2 = 120$

Therefore, the number of bacteria at the end of 2nd hour will be 120.

$a_5 = ar^4 = (30)(2)^4 = 480$

The number of bacteria at the end of 4th hour will be 480.

$a_{n+1} = ar^n = (30)\, 2^n$

Thus, number of bacteria at the end of nth hour will be $30(2)^n$.

Question 31:

What will Rs 500 amounts to in 10 years after its deposit in a bank which pays annual interest rate of 10% compounded annually?

The amount deposited in the bank is Rs 500.

At the end of first year, amount = $\text{Rs } 500\left(1+\frac{1}{10}\right)$ = Rs 500 (1.1)

At the end of 2nd year, amount = Rs 500 (1.1) (1.1)

At the end of 3rd year, amount = Rs 500 (1.1) (1.1) (1.1) and so on

∴Amount at the end of 10 years = Rs 500 (1.1) (1.1) … (10 times)

= Rs $500(1.1)^{10}$

Question 32:

If A.M. and G.M. of roots of a quadratic equation are 8 and 5, respectively, then obtain the quadratic equation.

Let the root of the quadratic equation be *a* and *b*.

According to the given condition,

$$\text{A.M.} = \frac{a+b}{2} = 8 \Rightarrow a+b = 16 \qquad \text{...(1)}$$
$$\text{G.M.} = \sqrt{ab} = 5 \Rightarrow ab = 25 \qquad \text{...(2)}$$

The quadratic equation is given by,

$x^2 - x$ (Sum of roots) + (Product of roots) = 0

$x^2 - x(a + b) + (ab) = 0$

$x^2 - 16x + 25 = 0$ [Using (1) and (2)]

Thus, the required quadratic equation is $x^2 - 16x + 25 = 0$

EXERCISE- 9.4

Question 1:

Find the sum to *n* terms of the series $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \ldots$

The given series is $1 \times 2 + 2 \times 3 + 3 \times 4 + 4 \times 5 + \ldots$

n^{th} term, $a_n = n(n + 1)$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k(k+1)$$

$$= \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k$$

$$= \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left(\frac{2n+1}{3}+1\right)$$

$$= \frac{n(n+1)}{2}\left(\frac{2n+4}{3}\right)$$

$$= \frac{n(n+1)(n+2)}{3}$$

Question 2:

Find the sum to n terms of the series $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \dots$

The given series is $1 \times 2 \times 3 + 2 \times 3 \times 4 + 3 \times 4 \times 5 + \dots$

n^{th} term, $a_n = n(n+1)(n+2)$

$= (n^2 + n)(n+2)$

$= n^3 + 3n^2 + 2n$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$
$$= \sum_{k=1}^{n} k^3 + 3\sum_{k=1}^{n} k^2 + 2\sum_{k=1}^{n} k$$
$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{3n(n+1)(2n+1)}{6} + \frac{2n(n+1)}{2}$$
$$= \left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$
$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + 2n + 1 + 2\right]$$
$$= \frac{n(n+1)}{2}\left[\frac{n^2 + n + 4n + 6}{2}\right]$$
$$= \frac{n(n+1)}{4}(n^2 + 5n + 6)$$
$$= \frac{n(n+1)}{4}(n^2 + 2n + 3n + 6)$$
$$= \frac{n(n+1)[n(n+2) + 3(n+2)]}{4}$$
$$= \frac{n(n+1)(n+2)(n+3)}{4}$$

Question 3:

Find the sum to n terms of the series $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots$

The given series is $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \ldots$

n^{th} term, $a_n = (2n + 1)\, n^2 = 2n^3 + n^2$

$$\therefore S_n = \sum_{k=1}^{n} a_k$$

$$= \sum_{k=1}^{n} = \left(2k^3 + k^2\right) = 2\sum_{k=1}^{n} k^3 + \sum_{k=1}^{n} k^2$$

$$= 2\left[\frac{n(n+1)}{2}\right]^2 + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n^2(n+1)^2}{2} + \frac{n(n+1)(2n+1)}{6}$$

$$= \frac{n(n+1)}{2}\left[n(n+1) + \frac{2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+2n+1}{3}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+5n+1}{3}\right]$$

$$= \frac{n(n+1)\left(3n^2+5n+1\right)}{6}$$

Question 4:

Find the sum to n terms of the series $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots$

The given series is $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \ldots$

n^{th} term, $a_n = \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$ (By partial fractions)

$$a_1 = \frac{1}{1} - \frac{1}{2}$$

$$a_2 = \frac{1}{2} - \frac{1}{3}$$

$$a_3 = \frac{1}{3} - \frac{1}{4} \ldots$$

$$a_n = \frac{1}{n} - \frac{1}{n+1}$$

Adding the above terms column wise, we obtain

$$a_1 + a_2 + \ldots + a_n = \left[\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \ldots \frac{1}{n}\right] - \left[\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots \frac{1}{n+1}\right]$$

$$\therefore S_n = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1} = \frac{n}{n+1}$$

Question 5:

Find the sum to n terms of the series $5^2 + 6^2 + 7^2 + \ldots + 20^2$

The given series is $5^2 + 6^2 + 7^2 + \ldots + 20^2$

n^{th} term, $a_n = (n+4)^2 = n^2 + 8n + 16$

$$\begin{aligned}\therefore S_n &= \sum_{k=1}^{n} a_k = \sum_{k=1}^{n}\left(k^2 + 8k + 16\right)\\ &= \sum_{k=1}^{n} k^2 + 8\sum_{k=1}^{n} k + \sum_{k=1}^{n} 16\\ &= \frac{n(n+1)(2n+1)}{6} + \frac{8n(n+1)}{2} + 16n\end{aligned}$$

16^{th} term is $(16+4)^2 = 20^2 2$

$$\begin{aligned}\therefore S_{16} &= \frac{16(16+1)(2\times16+1)}{6} + \frac{8\times16\times(16+1)}{2} + 16\times16\\ &= \frac{(16)(17)(33)}{6} + \frac{(8)\times16\times(16+1)}{2} + 16\times16\\ &= \frac{(16)(17)(33)}{6} + \frac{(8)(16)(17)}{2} + 256\\ &= 1496 + 1088 + 256\\ &= 2840\end{aligned}$$

$$\therefore 5^2 + 6^2 + 7^2 + \ldots\ldots + 20^2 = 2840$$

Question 6:

Find the sum to n terms of the series 3 × 8 + 6 × 11 + 9 × 14 +…

The given series is 3 × 8 + 6 × 11 + 9 × 14 + …

a_n = (n^{th} term of 3, 6, 9 …) × (n^{th} term of 8, 11, 14, …)

= $(3n)(3n + 5)$

= $9n^2 + 15n$

$$\begin{aligned}
\therefore S_n &= \sum_{k=1}^{n} a_k = \sum_{k=1}^{n}\left(9k^2+15k\right) \\
&= 9\sum_{k=1}^{n} k^2 + 15\sum_{k=1}^{n} k \\
&= 9\times\frac{n(n+1)(2n+1)}{6} + 15\times\frac{n(n+1)}{2} \\
&= \frac{3n(n+1)(2n+1)}{2} + \frac{15n(n+1)}{2} \\
&= \frac{3n(n+1)}{2}(2n+1+5) \\
&= \frac{3n(n+1)}{2}(2n+6) \\
&= 3n(n+1)(n+3)
\end{aligned}$$

Question 7:

Find the sum to n terms of the series $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^2) + \ldots$

The given series is $1^2 + (1^2 + 2^2) + (1^2 + 2^2 + 3^3\ ) + \ldots$

$a_n = (1^2 + 2^2 + 3^3 + \ldots\ldots. + n^2)$

$$=\frac{n(n+1)(2n+1)}{6}$$

$$=\frac{n\left(2n^2+3n+1\right)}{6}=\frac{2^3+3n^2+n}{6}$$

$$=\frac{1}{3}n^3+\frac{1}{2}n^2+\frac{1}{6}n$$

$$\therefore S_n=\sum_{k=1}^{n}a_k$$

$$=\sum_{k=1}^{n}\left(\frac{1}{3}k^3+\frac{1}{2}k^2+\frac{1}{6}k\right)$$

$$=\frac{1}{3}\sum_{k=1}^{n}k^3+\frac{1}{2}\sum_{k=1}^{n}k^2+\frac{1}{6}\sum_{k=1}^{n}k$$

$$=\frac{1}{3}\frac{n^2(n+1)^2}{(2)^2}+\frac{1}{2}\times\frac{n(n+1)(2n+1)}{6}+\frac{1}{6}\times\frac{n(n+1)}{2}$$

$$=\frac{n(n+1)}{6}\left[\frac{n(n+1)}{2}+\frac{(2n+1)}{2}+\frac{1}{2}\right]$$

$$=\frac{n(n+1)}{6}\left[\frac{n^2+n+2n+1+1}{2}\right]$$

$$=\frac{n(n+1)}{6}\left[\frac{n^2+n+2n+2}{2}\right]$$

$$=\frac{n(n+1)}{6}\left[\frac{n(n+1)+2(n+1)}{2}\right]$$

$$=\frac{n(n+1)}{6}\left[\frac{(n+1)(n+2)}{2}\right]$$

$$=\frac{n(n+1)^2(n+2)}{12}$$

Question 8:

Find the sum to n terms of the series whose n^{th} term is given by $n\ (n+1)\ (n+4)$.

$a_n = n\ (n+1)\ (n+4) = n(n^2+5n+4) = n^3+5n^2+4n$

$$\therefore S_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} k^3 + 5\sum_{k=1}^{n} k^2 + 4\sum_{k=1}^{n} k$$

$$= \frac{n^2(n+1)^2}{4} + \frac{5n(n+1)(2n+1)}{6} + \frac{4n(n+1)}{2}$$

$$= \frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{5(2n+1)}{3} + 4\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+3n+20n+10+24}{6}\right]$$

$$= \frac{n(n+1)}{2}\left[\frac{3n^2+23n+34}{6}\right]$$

$$= \frac{n(n+1)(3n^2+23n+34)}{12}$$

Question 9:

Find the sum to n terms of the series whose n^{th} terms is given by $n^2 + 2^n$

$a_n = n^2 + 2^n$

$$\therefore S_n = \sum_{k=1}^{n} k^2 + 2^k = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} 2^k \qquad (1)$$

Consider $\sum_{k=1}^{n} 2^k = 2^1 + 2^2 + 2^3 + \dots$

The above series 2, 2^2, 2^3, … is a G.P. with both the first term and common ratio equal to 2.

$$\therefore \sum_{k=1}^{n} 2^k = \frac{(2)\left[(2)^n - 1\right]}{2-1} = 2\left(2^n - 1\right) \qquad (2)$$

Therefore, from (1) and (2), we obtain

$$S_n = \sum_{k=1}^{n} k^2 + 2\left(2^n - 1\right) = \frac{n(n+1)(2n+1)}{6} + 2\left(2^n - 1\right)$$

Question 10:

Find the sum to n terms of the series whose n^{th} terms is given by $(2n - 1)^2$

$a_n = (2n - 1)^2 = 4n^2 - 4n + 1$

$$
\begin{aligned}
\therefore S_n &= \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \left(4k^2 - 4k + 1\right) \\
&= 4\sum_{k=1}^{n} k^2 - 4\sum_{k=1}^{n} k + \sum_{k=1}^{n} 1 \\
&= \frac{4n(n+1)(2n+1)}{6} - \frac{4n(n+1)}{2} + n \\
&= \frac{2n(n+1)(2n+1)}{3} - 2n(n+1) + n \\
&= n\left[\frac{2\left(2n^2 + 3n + 1\right)}{3} - 2(n+1) + 1\right] \\
&= n\left[\frac{4n^2 + 6n + 2 - 6n - 6 + 3}{3}\right] \\
&= n\left[\frac{4n^2 - 1}{3}\right] \\
&= \frac{n(2n+1)(2n-1)}{3}
\end{aligned}
$$